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OPTIMIZATION OF THE STRUCTURE OF DISTRIBUTION NETWORK BASED ON THEORIES OF INFORMATION, RELIABILITY AND PROBABILITY

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This article discusses approaches to the optimization of such technical systems as distribution networks. These approaches combine different fields of knowledge, including information theory, optimization methods, reliability theory, graph theory and probability theory. The prospects of application of this approaches and methods developed based on these approaches as a decision support system for designers of distribution networks of various kinds are revealed. An example of optimizing the structure of an arbitrary distribution network, such as an electric network, based on these approaches is presented.

Keywords: information uncertainty measure, structural reliability, reliability theory, information theory, probability theory, optimization methods.

ОПТИМИЗАЦИЯ СТРУКТУРЫ РАСПРЕДЕЛИТЕЛЬНОЙ СЕТИ НА БАЗЕ ТЕОРИЙ ИНФОРМАЦИИ, НАДЕЖНОСТИ И ВЕРОЯТНОСТЕЙ

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В данной статье рассматриваются подходы к оптимизации такого рода технических систем, как распределительные сети. Данные подходы сочетают в себе разные области знаний, в том числе теорию информации, методы оптимизации, теорию надежности, теорию графов и теорию вероятностей. Выявляется перспективность применения данного рода подходов и разработанных на базе данных подходов методов в качестве системы поддержки принятия решений для проектировщиков распределительных сетей различного рода. Представлен пример оптимизации на основе данных подходов структуры такой произвольной распределительной сети, как электрическая сеть.

Ключевые слова: мера неопределенности информации, структурная надежность, теория надежности, теория информации, теория вероятности, методы оптимизации.

Introduction. The development of new methods for the analysis and improvement of the design quality of optimal structures of technical systems of various kinds is quite an urgent task due to the fact that the fate of the built system will depend on the final version of the project. Most often, during the design process, designers pay more attention to minimizing the economic costs of building the system, as well as to some extent take into account the criteria of reliability and safety. However, taking into account the reliability criterion causes some difficulties due to the ambiguity of this concept [1]. There are a number of methods for calculating the reliability of the system [2]. At the same time, one of the promising approaches to the distinction between this indicator and its quantitative expression is the use of information theory tools [3].

Measure of information uncertainty in the problem of network reliability analysis. The application of the measure of information uncertainty in the problems of distinction of qualitative indicators of structural reliability is justified by the subjecting the occurrence of failures of technical system elements to the distribution laws of random variables, depending on the type of system, for example, the exponential law or Weibull's distribution [4], the density of which has the form:

$$f(x)=\left\{\frac{k}{\lambda}\cdot\left(\frac{x}{\lambda}\right)^{k-1}e^{-\left(\frac{x}{\lambda}\right)^{k}}\right. .$$

In works [5, 6], approaches for applying the measure of information uncertainty in problems of calculating structural reliability of a distribution network based on Claude Shannon's approaches [7] are offered. Claude Shannon's formula has the following form:

$$I=-\sum_{i=1}^{n}p_i\log p_i\ , \qquad (1)$$

by $\sum_{i=1}^{n}p_i=1,$

where, I – the amount of information; p_i – the probability of occurrence of the event i=1,2,...n.

The results obtained in these studies speak in favor of applying the measure of information uncertainty measures to solve problems in the field of ensuring the reliability of distribution networks, as well as the finding the optimal structures of designed networks. Let's consider the advantages of applying the measure of information uncertainty on a specific example. The classical method of calculating the reliability of the network is most often based on the exponential law of distribution of random variables [8].

$$p(t)=e^{-\int_0^t\lambda(t)dt}\approx e^{-\lambda t}=e^{-\frac{1}{t_{cp}}t}=e^{-\frac{t}{t_{cp}}}; \qquad (2)$$

where, t_{cp} – mean time between failures, t – the planned operating time of the element.

Let's suggest that a power line is non-operable state for an average of 35 hours per year, in this case, the probability of the operable state of the power line, for example, 100 hours in a row for, will be as following:

$$p(t)=e^{-\lambda\cdot t}=e^{-\frac{t}{t_{cp}}}=e^{-\frac{100}{8760-35}}=0{,}989$$

$$H(p)=-p\log_2 p=-0{,}989\log_2 0{,}989=0{,}0157\text{ bit.}$$

With 28 hours of being in a non-operable state line for the year:

$$p(t)=e^{-\lambda\cdot t}=e^{-\frac{t}{t_{cp}}}=e^{-\frac{100}{8760-28}}=0{,}988$$

$$H(p)=-p\log_2 p=-0{,}989\log_2 0{,}989=0{,}0172\text{ bit.}$$

As can be seen with a decrease in the structural reliability of the power line, the amount of information entropy tends to the maximum, while this maximum is achieved in almost equally probable states of operable and non-operable states. This situation is unlikely in reality, since already at a significant value of the probability of the non-operable state of the power line, a number of works on their modernization and replacement, in order to reduce the number of failures of the power lines will be performed. The disadvantages of the classical reliability calculation are the excessive scalability of the calculation formulas in the case of calculating the reliability of complex systems.

Optimization problem. The problem of finding the optimal structure of the distribution network, formulated in [9], includes an objective function and a system of constraints. The objective function of this optimization problem is as follows:

$$C(x_j) = \sum_{j=1}^{n} v_j x_j \rightarrow \min, \quad (3)$$

where v – the value of cost of the item x.

The constraints is a system of inequalities:

$$\begin{cases} H(P_1) \le H^0(P_1); \\ H(P_2) \le H^0(P_2); \\ \dots\dots\dots\dots\dots\dots \\ H(P_n) \le H^0(P_n); \end{cases} \quad (4)$$

where $H^0(P_1)$ – information entropy, the quantitative value of which reflects the requirement to ensure the required level of reliability of power supply to the consumer; $H(P_n)$ – information entropy of the connection between the energy source and the consumer, n – the number of the consumers.

The optimization problem (3) - (4) belongs to the class of integer combinatorial problems. This is justified by the fact that with the increase in the dimension and structural complexity of the system, increasing requirements for reliability and cost of solutions, the choice of optimal topology of a large network, in which the elements are ranked in according to their characteristics (the degree of importance of the network elements, the probability of failures of the network elements and their cost), is a non-trivial, combinatorial problem, often with conflicting and poorly formalized requirements. To solve this problem, you can firstly present the distribution network in an easy-to-analyze form, for example, using graph theories, to present it as a graph. Secondly, to determine the most optimal method of optimization (to solve this kind of problem) - one of the best methods for solving this kind of problems is the branch and bound method.

Branch and bound method. The branch and bound method was first proposed in 1960 by A. H. Land and A. G. Doig [10] for solving integer programming problems. This is a general algorithmic method for finding optimal solutions to various optimization problems, especially discrete and combinatorial optimization, finding the optimal solution and using the results of solving estimation problems. The branch and bound method is an algorithm of directed search of a set of solutions to the problem. The essence of the method is to restrict the complete enumeration by dividing the entire set of solutions into subsets and cutting off obviously unpromising subsets. If the lower bound of the values of the objective function in a subset is greater than the upper bound in the subset, then this subset can be omitted, since at least it is not achieved.

The algorithm of the branch and bound method. 1. First, the problem is solved without taking into account the integrality by linear programming methods, depending on the problem. 2. Specifying a set with additional constraints. 3. Calculation of estimates (boundaries). 4. Finding solutions to two problems with constraints. 5. In case of non-satisfaction of the integer conditions, branching and repetition of stages are carried out. The absence of a solution indicates that the problem is unsolvable.

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The mathematical solution by the branch and bound method is as follows:

$$\max(or\min)F=\sum_{j=1}^{n}c_jx_j;\ \ \sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}x_j=b_i;\ \ i=\overline{1,m};\ \ j=\overline{1,n};\ \ x_j\geq 0;$$

where x_j – integer.

Example of finding the optimal structure by the branch and bound method. Suppose we need to find the optimal structure of the simplest bridge network with 1 power source and 3 consumers. (fig. 1).

The scheme has the form of a complete undirected graph, i.e. there are 6 power lines and 2^6 = 64 variants of structures from which to choose the most optimal. At the same time, each line *i* has a cost (v_i) taken from the price list of stores and the probability of failure-free operation obtained during the analysis of statistical data on the work of similar types of lines in the past years. Each consumer has a limit value of the probability of operable state.

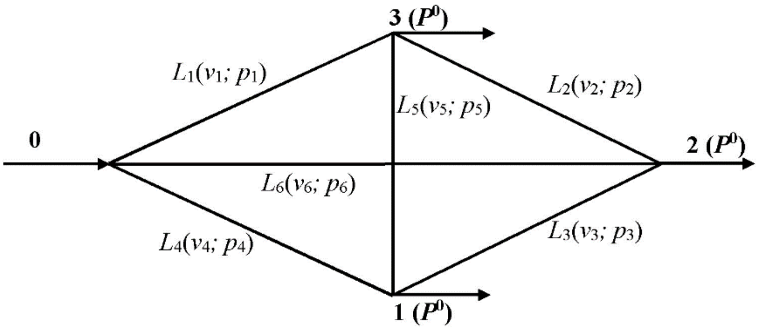

Fig. 1 – Distribution network

Set the values of all parameters: L_1(12; 0.984), L_2(15; 0.953), L_3(10; 0.986), L_4(13; 0.991), L_5(8; 0.978), L_6(18; 0.932); $P_1^0=0.983,\ P_2^0=0.974,\ P_3^0=0.967.$

Optimization problem:

$$C(x_i)=\sum_{i=1}^{n}v_ix_i\rightarrow \min,$$

$$\begin{cases}H(P_1)\leq H^0(P_1)\\ H(P_2)\leq H^0(P_2)\\ H(P_3)\leq H^0(P_3).\end{cases}$$

In the beginning, we find the cheapest of all structures by the method of branch and bound, for this simple structure, you can use the Prima algorithm, as a result, the cheapest structure $C(x_i)$ = 30 thousand dollars:

Next, check compliance with the restrictions:

$$\begin{cases}H(P_1)\leq H^0(0.983)\\ H(P_2)\leq H^0(0.974)\\ H(P_3)\leq H^0(0.967)\end{cases} => \begin{cases}H(P_1)\leq -0.983\cdot\log_2(0.983)=0.0243\text{ bit}\\ H(P_2)\leq -0.974\cdot\log_2(0.974)=0.037\text{ bit}\\ H(P_3)\leq -0.967\cdot\log_2(0.967)=0.0468\text{ bit}\end{cases}$$

$$H(P_1)=-p_1\cdot\log_2(p_5)-p_5\cdot\log_2(p_1)=-0.984\cdot\log_2(0.978)-0.978\cdot\log_2(0.984)$$
$$=0.0543>0.0243\text{ bit},$$

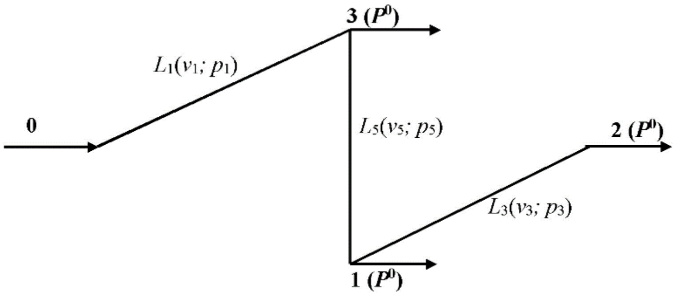

Fig. 2 – The cheapest structure

$$H(P_2) = -p_1 p_3 \cdot \log_2(p_5) - p_3 p_5 \cdot \log_2(p_1) - p_1 p_5 \cdot \log_2(p_2) =$$
$$= -0.984 \cdot 0.986 \cdot \log_2(0.978) - 0.986 \cdot 0.978 \cdot \log_2(0.984) -$$
$$- 0.984 \cdot 0.978 \cdot \log_2(0.986) = 0.128 >> 0.037 \text{bit},$$
$$H(P_3) = -p_1 \cdot \log_2(p_1) = -0.984 \cdot \log_2(0.984) = 0.0228 < 0.0468 \text{ bit}.$$

To increase the level of reliability of consumers 1 and 2, we add line 4 to the structure, since it is closer to the power source, and check whether this has allowed to increase the level of reliability to the required value:

$$H(P_1) = -((1 - p_1 \cdot p_5) \cdot \log_2(p_5) + (1 - p_1 \cdot p_5) \cdot \log_2(p_1)) - p_4 \cdot \log_2(p_4) = 0{,}00044 <$$
$$< 0.0243 \text{ bit},$$
$$H(P_2) = H(P_1) - p_3 \cdot \log_2(p_3) = 0.0204 < 0.037 \text{ bit}.$$

Thus, the optimal network structure will have the form shown in figure 3:

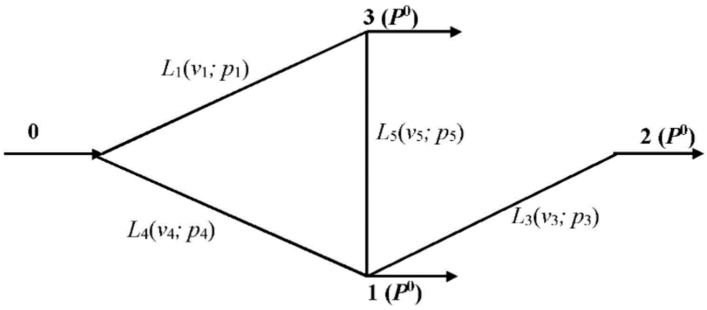

Fig. 3 – The optimal structure of the distribution network

Result:

$$C(x_i) = \sum_{i=1}^{n} v_i x_i = 43000\$,$$

$$\begin{cases} H(P_1) \le H^0(P_1) \\ H(P_2) \le H^0(P_2) \\ H(P_3) \le H^0(P_3) \end{cases} => \begin{cases} 0.00044 < 0.243 \\ 0.0204 < 0.037 \\ 0.0228 < 0.0468. \end{cases}$$

Conclusion. The foregoing evidence in favor of applying the measures of information uncertainty in the task of delineating the structural reliability of the distribution network in quantitative form. Classical methods of reliability calculation don’t allow to monitor the degree of information change in the network structure, and formulas for calculating system reliability will be excessively massive to calculate too complex systems, which can affect information processing time and the decision to reserve the least reliable parts of the network. All this speaks in favor of further research and study of the possibility of applying the measure of information uncertainty for the system analysis of distribution network structures.

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